# PRAYAS

## JEE 2026

### Mathematics

# Basic Maths

Lecture -15

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# Topics *to be covered*

**A** Inequalities involving Modulus

**B** Problem Practice

# Homework Discussion

**QUESTION**

★★★★KCLS★★★★

**KTK 04**

If $(x_1, y_1)$ and $(x_2, y_2)$ are the solution of the system of equation.

$\log_{225}(x) + \log_{64}(y) = 4$

$\log_x(225) - \log_y(64) = 1,$

then show that the value of $\log_{30}(x_1y_1x_2y_2) = 12.$

let $\log_{225} x = a$, $\log_{64} y = b$

$$a+b=4 \Rightarrow b=4-a$$

$$\frac{1}{a}-\frac{1}{b}=1$$

$$\frac{1}{a}-\frac{1}{4-a}=1$$

$$4-a-a=a(4-a)$$

$$4-2a=4a-a^2$$

$$a^2-6a+4=0 \begin{cases} a_1=\log_{225} x_1 \\ a_2=\log_{225} x_2 \end{cases}$$

$$a_1+a_2=\log_{225} x_1\cdot x_2$$

$$6=\log_{225} x_1x_2 \rightarrow x_1x_2=(225)^6$$

$$\log_{64} y_1 = b_1 = 4-a_1$$

$$\log_{64} y_2 = b_2 = 4-a_2$$

$$\log_{64} y_1\cdot y_2 = 8-(a_1+a_2)$$

$$\log_{64}(y_1 \cdot y_2) = 8-6=2$$

$$y_1 y_2 = 64^2$$

$$\log_{30}(x_1 x_2 y_1 y_2) = \log_{30}\left((225)^6 \cdot 64^2\right)$$

$$= \log_{30}(15^{12} \cdot 2^{12}) = \log_{30} 30^{12} = 12\log_{30} 30 = 12$$ **Ans**

**QUESTION**

★★★ KCLS ★★★★ KTK 05

The sum of the roots of the equation $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is :

(A) $\log_2 14$

(B) $\log_2 11$

(C) $\log_2 12$

(D) $\log_2 13$

$$\log_2 2^{x+1} - 2\log_2(3+2^x) + \frac{2}{2}\log\left(10-\frac{1}{2^x}\right) = 0$$

$$\log_2\left(\frac{2^{x+1}\cdot\left(10-\frac{1}{2^x}\right)}{(3+2^x)^2}\right) = 0$$

$$2^{x+1}\left(10-\frac{1}{2^x}\right) = 2^0\cdot(3+2^x)^2$$

let $2^x = t$

$$2\cdot t\left(10-\frac{1}{t}\right) = (3+t)^2$$

$$20t-2 = 9+t^2+6t$$

$$t^2-14t+11=0 \left\langle \begin{matrix} t_1 = 2^{x_1} \\ t_2 = 2^{x_2} \end{matrix} \right\rangle \begin{matrix} t_1\cdot t_2 = 2^{x_1+x_2} \\ 11 = 2^{x_1+x_2} \end{matrix}$$

Ans. B

$$2^{x_1+x_2} = 11$$

$$\log_2 2^{x_1+x_2} = \log_2 11$$

$$(x_1 + x_2) = \log_2 11 .$$

# Home Challenge-06

$x>0$ , $x-1>0$

✓ $x>1$ clearly.

If $x = \alpha$ is the solution of the equation $|2 + \log_2 7x| - \log_2(x - 1) = 5$, then find the value of $(65)^{\frac{1}{3}\log_{\alpha^2+1}\alpha}$. [Ans. 2]

Case ① $2+\log_2 7x \geq 0 \Rightarrow \log_2 7x \geq -2 \Rightarrow 7x \geq 2^{-2} = \frac{1}{4}$

$x \geq \frac{1}{28}$

$2+\log_2 7x - \log_2(x-1) = 5$

$\log_2 \frac{7x}{x-1} = 3$

$\frac{7x}{x-1} = 8 \Rightarrow 7x = 8x-8$

$\boxed{x=8}$

case (II) If $2+\log_2 7x < 0$

$x < \frac{1}{28}$ (N.P)

Ans: $x = 8 = \alpha$

$65^{\frac{1}{3}\log_{65} 8} = 8^{1/3} = 2$ Ans

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

$\boxed{\text{bpp} \to 4}$

Solution:-

$$\boxed{\log_{(1/2)} \log_5 (\log_2 (x^2-6x+40))) > 0}$$

$\log_5 (\log_2 (x^2-6x+40)) > 1$ ?? , $\log_5 (\log_2 (x^2-6x+40)) > 0$, $\log_2 (x^2-6x+40) > 0$

↓ No Need

↓ No Need

(sign change)

$\log_2 (x^2-6x+40) > 5$

$x^2-6x+40 > 32$

$x^2-6x+8 > 0$

$(x-4)(x-2) > 0$

$x \in (-\infty, 2) \cup (4, \infty)$

$x^2-6x+40 > 0$

↓

No Need

Galti Batao

Question:- Find the integral value of x satisfying the equation

$$\Rightarrow |\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$$

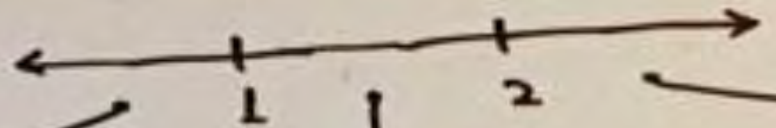
$$|2\log_3 x - 2| - |\log_3 x - 2| = 2$$

Let:- $\log_3 x = t$

$$|2t - 2| - |t - 2| = 2$$

$$2|t-1| - |t-2| - 2 = 0$$

TAH-2



$x \leq 1$

$1 < x < 2$

$x \geq 2$

**Galti Batao**

$-2t + 2 + t - 2 - 2 = 0$

$-t - 2 = 0$

$\boxed{t = -2}$ accepted

$\log_3 x = -2$

$x = 3^{-2}$

$\boxed{x = \frac{1}{9}}$

$x = 0.11\ldots$

$2t - 2 + t - 2 - 2 = 0$

$3t - 6 = 0$

$\boxed{t = 2}$

rejected

$2t - 2 - t + 2 - 2 = 0$

$t - 2 = 0$

$\boxed{t = 2}$ accepted

$\log_3 x = 2$

$x = 3^2$

$\boxed{x = 9}$

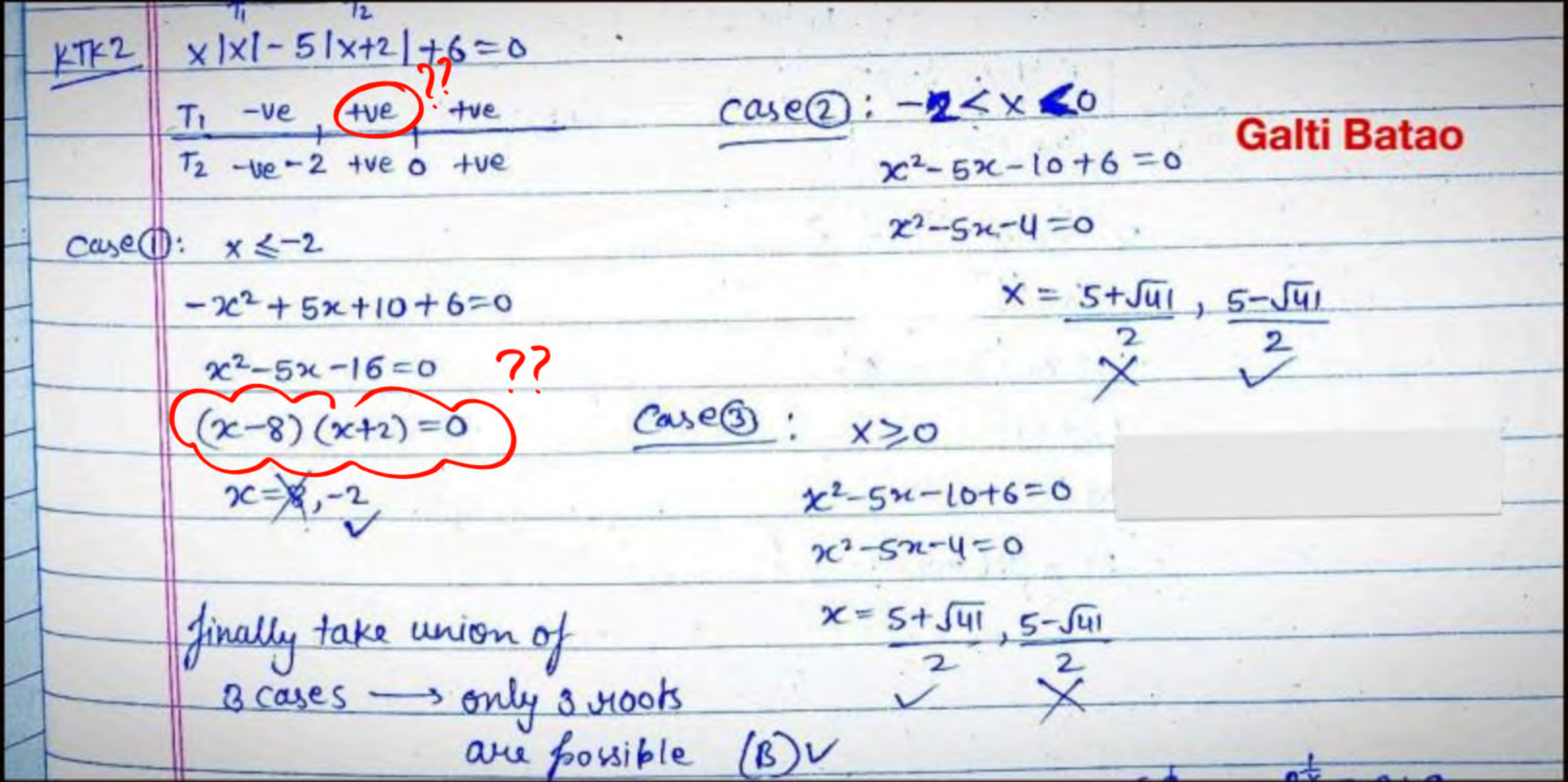
$\cup$

$(0.1, 9)$

???

$x = 1/9, 9$

integral value is $\Rightarrow \underbrace{1, 2, 3, 4, 5, 6, 7, 8, 9}_{9}$



**Galti Batao**

KTK2 $\quad x\overset{T_1}{|x|} - 5\overset{T_2}{|x+2|} + 6 = 0$

$T_1$ −ve , $\boxed{\text{+ve}}$ ?? +ve

$T_2$ −ve −2 +ve 0 +ve

Case①: $x \leq -2$

$-x^2 + 5x + 10 + 6 = 0$

$x^2 - 5x - 16 = 0$ ??

$\boxed{(x-8)(x+2) = 0}$

$x = \cancel{8}, -2$ ✓

Case②: $-2 < x < 0$

$x^2 - 5x - 10 + 6 = 0$

$x^2 - 5x - 4 = 0$

$x = \frac{5+\sqrt{41}}{2}, \frac{5-\sqrt{41}}{2}$

✗ ✓

Case③: $x \geq 0$

$x^2 - 5x - 10 + 6 = 0$

$x^2 - 5x - 4 = 0$

$x = \frac{5+\sqrt{41}}{2}, \frac{5-\sqrt{41}}{2}$

✓ ✗

finally take union of 3 cases ⟶ only 3 roots are possible (B)✓

**QUESTION**

Values of x for which $f(x) = \log_{|x-3|}\left(\frac{|x|-5}{2-|x|}\right)$ is defined.

$|x-3|>0$ & $|x-3|\neq 1$ & $\frac{|x|-5}{2-|x|}>0$

↓

$x\neq 3$

$x-3\neq \pm 1$

$x\neq 3\pm 1$

$x\neq 4,2$

M① $|x|=t$ (let) OR M②

$\frac{t-5}{2-t}>0$

$\frac{t-5}{t-2}<0$

$t\in(2,5)$

$2<|x|<5$

$x\in(-5,-2)\cup(2,5)$

∩

Ans: $(-5,-2)\cup(2,5)-\{3,4\}$

case① $x\geq 0$

$\frac{x-5}{2-x}>0$

$\frac{x-5}{x-2}<0$

$x\in(2,5)$

∩ $x\in(2,5)$

case(ii) if $x<0$

$\frac{-x-5}{2+x}>0$ ∩ $x\in(-5,-2)$

$\frac{x+5}{x+2}<0$ — $x\in(-5,-2)$

$x\in(-5,-2)\cup(2,5)$

Ans. $x \in (-5,-2)\cup(2,5)-\{3,4\}$

## QUESTION [JEE Mains 2022 (28 July)]

★★★ KCLS ★★★

Let $S = \left\{x \in [-6,3] - \{-2,2\} : \frac{|x+3|-1}{|x|-2} \geq 0\right\}$ and $T = \{x \in \mathbb{Z} : x^2 - 7|x| + 9 \leq 0\}$.

Then the number of elements in $S \cap T$ is :

(A) 7

(B) 5

(C) 4

(D) 3

$$\frac{\overset{T_1}{|x+3|-1}}{\underset{T_2}{|x|-2}} \geq 0$$

-ve +ve +ve

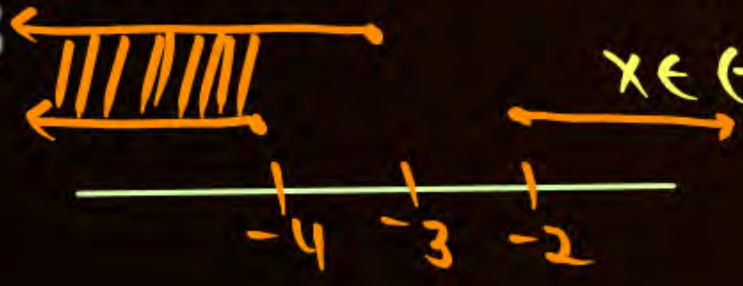
-ve -3 -ve 0 +ve

case ① if $x \leq -3$

$$\frac{-x-3-1}{-x-2} \geq 0 \quad \cap$$

$$\frac{x+4}{x+2} \geq 0$$

$x \in (-\infty,-4] \cup (-2,\infty)$



$x \in (-\infty,-4]$ ✓

case(ii) if $-3 < x < 0$

$$\frac{x+3-1}{-x-2} \geq 0$$

$$\frac{x+2}{x+2} \leq 0$$

$1 \leq 0$ , $x \neq -2$

(N.P) ⇓ $x \in \phi$ ✓

case(iii) if $x \geq 0$

$$\frac{x+3-1}{x-2} \geq 0$$

$$\frac{x+2}{x-2} \geq 0 \Rightarrow x \in (-\infty,-2] \cup (2,\infty)$$

$\cap$ $x \in (2,\infty)$ ✓

Ans. D

$x \in (-\infty,-4] \cup (2,\infty) \rightarrow [-6,-4] \cup (2,3] = S$

But for S $x \in [-6,3] - \{-2,2\}$

for T

$$\boxed{|x|^2 = x^2}$$

$$x^2 - 7|x| + 9 \le 0$$

$$|x|^2 - 7|x| + 9 \le 0$$

$$|x| = \frac{7 \pm \sqrt{13}}{2}$$

$$\left(|x| - \frac{7+\sqrt{13}}{2}\right)\left(|x| - \frac{7-\sqrt{13}}{2}\right) \le 0$$

$$|x| \in \left[\frac{7-\sqrt{13}}{2}, \frac{7+\sqrt{13}}{2}\right]$$

$$\frac{7-\sqrt{13}}{2} < |x| < \frac{7+\sqrt{13}}{2}$$

$R^+$

$$|x| \in [a,b]$$

$$x \in [-b,-a] \cup [a,b]$$

$$x \in \left[\overset{-5...}{-\left(\frac{7+\sqrt{13}}{2}\right)}, \overset{-1.5...}{-\left(\frac{7-\sqrt{13}}{2}\right)}\right] \cup \left[\overset{1.5...}{\frac{7-\sqrt{13}}{2}}, \overset{5...}{\frac{7+\sqrt{13}}{2}}\right]$$

$x \in Z \Rightarrow T = \{-5,-4,-3,-2,2,3,4,5\}$

$S \cap T = \{-5, -4, 3\}$

## QUESTION

Solve: $|x^2 - x - 6| \le x^2 + x - 10$

case① If $x^2 - x - 6 \ge 0 \rightarrow (x-3)(x+2) \ge 0 \rightarrow x \in (-\infty, -2] \cup [3, \infty)$

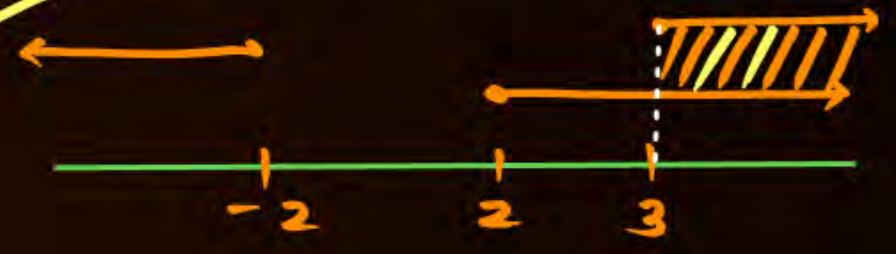
$\cancel{x^2} - x - 6 \le \cancel{x^2} + x - 10$

$2x \ge 4$

$x \ge 2$.

$\cap$

$x \in [3, \infty)$



Case (II) If $x^2 - x - 6 < 0 \rightarrow x \in [-2, 3)$

$-x^2 + x + 6 \le x^2 + x - 10$

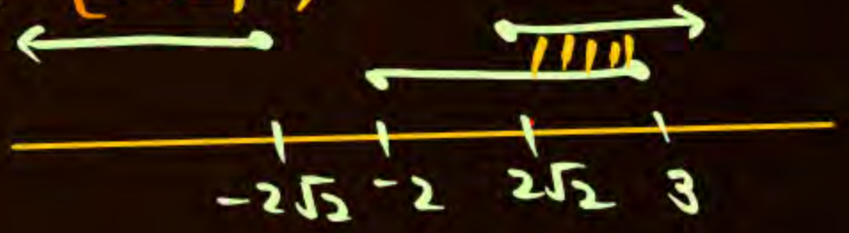
$2x^2 - 16 \ge 0$

$x^2 - (2\sqrt{2})^2 \ge 0$

$(x - 2\sqrt{2})(x + 2\sqrt{2}) \ge 0$

$x \in (-\infty, -2\sqrt{2}] \cup [2\sqrt{2}, \infty)$

$x \in [2\sqrt{2}, 3)$



$\cup$

$x \in [2\sqrt{2}, 3) \cup [3, \infty)$

$x \in [2\sqrt{2}, \infty)$

**QUESTION**

If $\left||x-2|-3\right| \le 2$ then complete set of values of x is

(A) $[-3, 7]$

[✓ (B)] $[-3, 1] \cup [3, 7]$

(C) $[1, 7]$

(D) $[-3, 3] \cup [4, 7]$

$-2 \le |x-2|-3 \le 2$

$1 \le |x-2| \le 5$

$|x-2| \ge 1$

$x-2 \le -1$ or $x-2 \ge 1$

$x \le 1$ or $x \ge 3$

$x \in (-\infty, 1] \cup [3, \infty)$

$|x-2| \le 5$

$-5 \le x-2 \le 5$

$-3 \le x \le 7$

$x \in [-3, 7]$

$\cap$

$[-3, 1] \cup [3, 7]$

$|x| \le a, \; a \in R^+$

$-a \le x \le a$

$|x| \ge a, \; a \in R^+$

$x \le -a$ or $x \ge a$

## QUESTION

$||x - 2| - 3| \leq 0$ then number of values of x satisfy the given inequality is

A 0

B 1

[✓ C] 2

D Infinite

$||x-2|-3| \leq 0$ But $||x-2|-3| \geq 0$

$\Rightarrow ||x-2|-3| = 0$

$|x-2|-3=0$

$|x-2|=3$

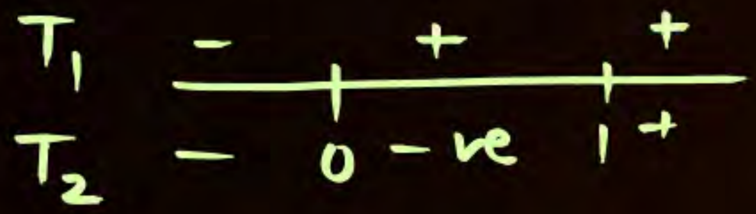
$x-2=-3,3$

$x=-1,5.$

**QUESTION**

$T_1$ $T_2$

Solve: $|x| + |x - 1| \geq 7$



Tahol

**QUESTION**

Solve: $|x+1|+|x-1|>2$

Tan02

## QUESTION

If $|x^2 - x - 10| > |x^2 - 11x - 22|$ then find the possible set of all values of x.

$|x| \geq |y|$

$\Rightarrow x^2 \geq y^2$

$$|x^2-x-10|^2 > |x^2-11x-22|^2$$

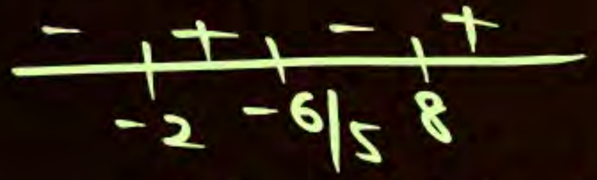
$$(x^2-x-10)^2 > (x^2-11x-22)^2$$

$$(x^2-x-10+x^2-11x-22)\,(x^2-x-10-x^2+11x+22)>0$$

$$(2x^2-12x-32)(10x+12)>0$$

$$(x^2-6x-16)(5x+6)>0$$

$$(x-8)(x+2)(5x+6)>0$$



$x\in(-2,-6/5)\cup(8,\infty)$

**QUESTION**

Solve: $|x - 6| > |x^2 - 5x + 9|$

Jan 03

QUESTION

Solve: $(|x-1|-3)(|x+2|-5)<0$

$|x-1|-3<0$ & $|x+2|-5>0$

$|x-1|<3$ & $|x+2|>5$

$-3<x-1<3$ & $x+2<-5$ or $x+2>5$

$-2<x<4$ & $x<-7$ or $x>3$

$x\in(-2,4)$ & $x\in(-\infty,-7)\cup(3,\infty)$

$\cap$

$x\in(3,4)$

or $|x-1|-3>0$ & $|x+2|-5<0$

$|x+2|<5$

$|x-1|>3$

$x-1<-3$ or $x-1>3$ & $-5<x+2<5$

$x<-2$ or $x>4$ & $-7<x<3$

$x\in(-\infty,-2)\cup(4,\infty)$ & $x\in(-7,3)$

$\cap$

$x\in(-7,-2)$

UNION

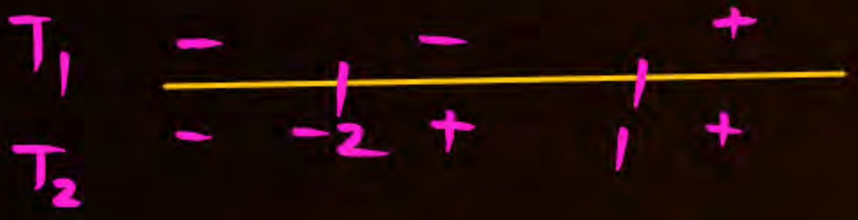
$x\in(-7,-2)\cup(3,4)$

## QUESTION

Solve: $(\overset{T_1}{|x-1|-3})(\overset{T_2}{|x+2|-5}) < 0$

Tan04

M②



# Using Triangle Inequality

$$P_9: \ \big||a| - |b|\big| \le |a+b| \le |a| + |b|$$

$$|a+b| = |a| + |b| \iff ab \geq 0$$

$$|a+b| = \big||a|-|b|\big| \iff ab \leq 0$$

Ex: $\big||2|-|-3|\big| \le |2+(-3)| \le |2| + |-3|$

$$1 \le \overset{\shortparallel}{1} \le 5$$

Ex: $\big||-5|-|-6|\big| \le |-5-6| \le |-5| + |-6|$

$$\overset{\shortparallel}{1} \le \overset{\shortparallel}{11} \le \overset{\shortparallel}{11}$$

$b \to -b$

$$\big||a|-|-b|\big| \le |a-b| \le |a| + |-b|$$

$$P_{10}: \ \big||a|-|b|\big| \le |a-b| \le |a| + |b|$$

$$|a-b| = |a| + |b| \iff a \cdot b \le 0$$

$$\big||a|-|b|\big| = |a-b| \iff ab \geq 0$$

**QUESTION**

Solve: $|\overset{a}{x - 2}| + |\overset{b}{x - 5}| = 3$

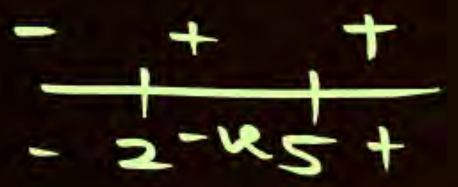
$|a|+|b| \geq |a-b|$

M① $3 = \underbrace{|x-2| + |x-5|} \geq |\underbrace{x-2-x+5}| = 3$

$(x-2)(x-5) \leq 0$

$x \in [2,5]$

M②



$|x-2| + |x-5| = 3$

Case ① $x \leq 2$

$-(x-2)-(x-5)=3$

$-x+2-x+5=3$

$2x=4$

$x=2$ ✓

Case (ii) $2<x<5$

$x-2+5-x=3$

$3=3 \rightarrow x \in R$

$\cap \rightarrow x \in (2,5)$

Case (ii) $x \geq 5$

$x-2+x-5=3$

$2x=10$

$x=5$ ✓

UNION

$x \in [2,5]$

M③ $|x-2| + |-(5-x)| = 3$

$|x-2| + |5-x| = 3$

$3 = |x-2| + |5-x| \geq |x-2+5-x| = 3$

$(x-2)(5-x) \geq 0$

$(x-2)(x-5) \leq 0$

$x \in [2,5]$

## QUESTION

Solve: $\underbrace{\left|\frac{x}{x-1}\right|}_{a} + \underbrace{|x|}_{b} = \frac{x^2}{|x-1|}$

observe!! $a+b = \frac{x}{x-1} + x = \frac{x+x^2-x}{x-1} = \frac{x^2}{x-1}$

$$|a| + |b| = |a+b|$$

$$\Downarrow$$

$$a \cdot b \geq 0.$$

$$\frac{x}{x-1} \cdot x \geq 0$$

$$\frac{x^2}{x-1} \geq 0$$

$\frac{1}{x-1} \geq 0$, $x = 0$ is also possible

$x-1 > 0$

$x > 1$ $\Rightarrow$ Ans: $x \in \{0\} \cup (1, \infty)$

## QUESTION

Solve: $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}=2$

$$\sqrt{x-1+1+2\sqrt{x-1}}+\sqrt{x-1+1-2\sqrt{x-1}}=2$$

$$\sqrt{\sqrt{x-1}^2+1^2+2\sqrt{x-1}\cdot 1}+\sqrt{\sqrt{x-1}^2+1^2-2\sqrt{x-1}}=2$$

$$|\sqrt{x-1}+1|+|\sqrt{x-1}-1|=2$$

$$2=|\sqrt{x-1}+1|+|\sqrt{x-1}-1|\geq|\sqrt{x-1}+1-(\sqrt{x-1}-1)|=2$$

$\underbrace{\text{By Triangle Ineq.}}$

$$(\sqrt{x-1}+1)(\sqrt{x-1}-1)\leq 0$$

$$(x-1-1)\leq 0$$

$$x\leq 2\rightarrow x\in(-\infty,2]\quad \text{Ans } (-\infty,2]\cap[1,\infty)$$

$$x\in\overset{\parallel}{[1,2]}$$

clearly $x-1\geq 0\rightarrow \boxed{x\geq 1}$

$x+2\sqrt{x-1}\geq 0$ ✓ True

$x-2\sqrt{x-1}\geq 0$

$x\geq 2\sqrt{x-1}$

$x^2\geq 4x-4$

$x^2-4x+4\geq 0$

$(x-2)^2\geq 0$ true.

**QUESTION**

Solve for values of x :

(i) $|3x-5| + |8-x| = |3+2x|$

(ii) $|x^2-5x+6| + |x^2-4| = 5|x-2|$

$|3x-5| + |8-x| \geq |3x-5+8-x| = |3+2x|$

By Triangle Ineq

$(3x-5)\cdot(8-x) \geq 0$

$(3x-5)(x-8) \leq 0$

$x \in [5/3, 8]$

$|x^2-5x+6| + |x^2-4| \geq |x^2-5x+6-x^2+4|$

$\| \quad |10-5x| = 5|2-x|$

$= 5|x-2|$

$\Downarrow$

$(x^2-5x+6)(x^2-4) \leq 0$

$(x-2)(x-3)(x-2)(x+2) \leq 0$

$(x-3)(x+2) \leq 0$ , $x=2$ is also possible

$x \in [-2,3]$ Ans

# QUESTION

Tanos

(a) $|x^2 - 9| + |16 - x^2| = 7$

Ans: $x \in [-4, -3] \cup [3, 4]$

(b) Solve: $|(x^2 + 2x + 2) + (3x + 7)| < |x^2 + 2x + 2| + |3x + 7|$

Ans: $x < -7/3$

$$\big||a|-|b|\big| = |a+b| \iff ab<0$$

$$|a+b| = |a|+|b| \iff ab \geq 0$$

$$\big||a|-|b|\big| \leq |a+b| \leq |a|+|b|$$

$$|a+b| < |a|+|b| \iff ab<0 .$$

$$\big||a|-|b|\big| < |a+b| \iff ab>0 .$$

$$\big||a|-|b|\big| = |a-b|$$
$$\Updownarrow$$
$$ab \geq 0$$

$$|a-b| = |a|+|b| \iff ab \leq 0$$

★ $$\big||a|-|b|\big| \leq |a-b| \leq |a|+|b|$$

$$|a-b| < |a|+|b| \iff ab>0 .$$

$$\big||a|-|b|\big| < |a-b| \iff ab<0$$

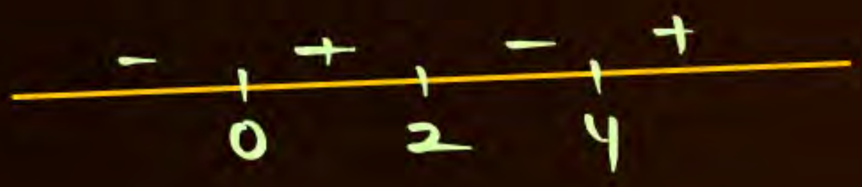
## QUESTION

Solve: $|x^2 - 2x| + |x - 4| > |x^2 - 3x + 4|$

$|a| + |b| > |a - b|$

$(x^2 - 2x) \cdot (x - 4) > 0$

$x(x-2)(x-4) > 0$



$x \in (0, 2) \cup (4, \infty)$

# Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

# Home Challenge-07

Find the sum of all the integral solution(s) of the equation $3^{|x|} = \left(\frac{3}{(\sqrt{3})^{|x-2|}}\right)^2$. [Ans. 3]

# Today's BPP

## Bumper Practice Problems

Solve the following inequality for x.

(a) $|x| > 2$

(b) $|x - 1| > 3$

(c) $|x - 2| < 1$

(d) $|x + 1| \geq 2$

(e) $|x - 1| \leq 5$

(f) $|2x - 3| > 7$

(g) $|3x + 5| < 2$

(h) $|4x + 6| > 5$

(i) $|2x - 3| > -2$

(j) $|4x - 9| \leq 7$

(k) $|3x + 5| \geq 2$

(l) $|2x + 3| \geq 0$

(m) $|4 - 3x| < -2$

(n) $|5 - 3x| \leq 0$

(o) $\left|\frac{x-3}{x-5}\right| > 1$

(p) $\left|\frac{x+4}{x+2}\right| \leq 1$

(q) $|x^2 - 4x| < 5$

(r) $|x - 3| > -1$

(s) $|3x - 2.5| \geq 2$

(t) $|x - 2| \leq |x + 4|$

## Answers

(a) $(-\infty, -2) \cup (2, \infty)$

(b) $(-\infty, -2) \cup (4, \infty)$

(c) $x \in (1, 3)$

(d) $x \in (-\infty, -3] \cup [1, \infty)$

(e) $x \in (-4, 6)$

(f) $(-\infty, -2) \cup (5, \infty)$

(g) $\left(-\frac{7}{3}, -1\right)$

(h) $x \in \left(-\infty, \frac{-11}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$

(i) $x \in (-\infty, \infty)$

(j) $x \in \left[\frac{1}{2}, 4\right]$

(k) $x \in \left(-\infty, \frac{-7}{3}\right] \cup [-1, \infty)$

(l) $x \in \mathbb{R}$

(m) $x \in \phi$

(n) $x \in \left\{\frac{5}{3}\right\}$

(o) $(4, \infty) - \{5\}$

(p) $(-\infty, -3]$

(q) $(-1, 5)$

(r) $x \in (-\infty, \infty)$

(s) $x \in \left(-\infty, \frac{1}{6}\right] \cup \left[\frac{3}{2}, \infty\right)$

(t) $x \in [-1, \infty)$

QUESTION

Solve the following equations

(a) $|x-1|+|x-3|=2$

(b) $|x|+|x+5|=5$

(c) $|x-1|+|x-4|=2$

(d) $|x^2-2x|+|x-4|=|x^2-3x+4|$

Answers:

(a) $[1,3]$ (b) $[-5,0]$ (c) $\{\}$ (d) $x \in (-\infty,0] \cup [2,4]$

# Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

(KTK 1)

| Column-I | Column-II |
|---|---|
| (A) $\dfrac{\log_2 32}{\log_3 \sqrt{243}}$ | (P) positive integer |
| (B) $\dfrac{2\log 6}{\log 12+\log 3}$ | (Q) negative integer |
| (C) $\log_{1/4}\left(\frac{1}{16}\right)^{-2}$ | (R) rational but not integer |
| (D) $\dfrac{\log_5 16-\log_5 4}{\log_5 128}$ | (S) Prime |

Ans. A → P, S; B → P; C → Q; D → R

**QUESTION**

(KTK 2)

$$\frac{1}{x-1}-\frac{4}{x-2}+\frac{4}{x-3}-\frac{1}{x-4}<\frac{1}{30}$$

Ans. $x \in (-\infty,-2) \cup (-1,1) \cup (2,3) \cup (4,6) \cup (7,\infty)$

**QUESTION**

(KTK 3)

The equation $\frac{\log_8\left(\frac{8}{x^2}\right)}{(\log_8 x)^2} = 3$ has

**A** no integral solution

**B** one natural solution

**C** two real solutions

**D** one irrational solution

Ans. B, C

QUESTION

(KTK 4)

For the equation $\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$, which of the following do not hold good?

A no real solution

B one prime solution

C one integral solutions

D no irrational solution

Ans. A, B, D

QUESTION

(KTK 5)

If $\log_{\sqrt{2}}\sqrt{x} + \log_2 x + \log_4(x^2) + \log_8(x^3) + \log_{16}(x^4) = 40$ then x is equal to

Ans. 256

**QUESTION**

(KTK 6)

| Column-I | | Column-II | |
|---|---|---|---|
| (A) | If $x_1$ and $x_2$ satisfy the equation $x^{\log_{10} x} = 100x$ then the value of $x_1 x_2$ equals | (P) | irrational |
| (B) | Sum of the squares of the roots of the equation $\log_2(9 - 2^x) = 3 - x$ is | (Q) | rational |
| (C) | If $\log_{\frac{1}{8}}\left(\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right)\right) = \frac{1}{3}$ then x is | (R) | prime |
| (D) | Let $\log_b a = 3, \log_b c = -4$. If the value of x satisfying the equation $a^{3x} = c^{x-1}$ is expressed in the form $p/q$, where p and q are relatively prime then $p + q$ is | (S) | composite |

Ans. A → Q, S; B → Q, S; C → P; D → QR

# Homework From Module

**Prarambh (Topicwise)** : Q1 to Q25

**Prabal (JEE Main Level)** : Q1 to Q33

# Solution to Previous KTKs

QUESTION

(KTK 1)

Solve for x : $3|x^2 - 4x + 2| = 5x - 4$

Ans. $x = 2, 5$

* KTK

1.

$$3|x^2-4x+2| = 5x-4$$

$x^2-4x+2 \geq 0$

$\frac{4 \pm \sqrt{16-8}}{2} \rightarrow 2 \pm \sqrt{2}$

$(-\infty, 2-\sqrt{2}] \cup [2+\sqrt{2}, \infty)$

$3x^2-12x+6 = 5x-4$

$3x^2-17x+10 = 0$

$3x^2-15x-2x+10 = 0$

$3x(x-5)-2(x-5) = 0$

$(3x-2)(x-5) = 0$

$x = \overset{\times}{\frac{2}{3}}, \overset{\checkmark}{5}$

$x^2-4x+2 \leq 0$

$(2-\sqrt{2}, 2+\sqrt{2})$

$-3x^2+12x-6 = 5x-4$

$3x^2-7x+2 = 0$

$3x^2-6x-x+2 = 0$

$3x(x-2)-1(x-2) = 0$

$(3x-1)(x-2) = 0$

$x = \overset{\times}{\frac{1}{3}}, 2 \checkmark$

$$\boxed{x = 2, 5}$$

## * KTK

KTK-1) Solve for $x$: $3|x^2-4x+2| = 5x-4$

Case-1:

$x^2-4x+2 \geq 0$

$x = \frac{4 \pm \sqrt{16-4(2)}}{2}$

$x = \frac{4 \pm 2\sqrt{2}}{2} \Rightarrow x = 2+\sqrt{2},\ 2-\sqrt{2}$

$(x-(2+\sqrt{2}))(x-(2-\sqrt{2})) \geq 0$

$x \in (-\infty, 2-\sqrt{2}] \cup [2+\sqrt{2}, \infty)$

$3(x^2-4x+2)-5x+4=0$

$3x^2-12x+6-5x+4=0$

$3x^2-17x+10=0$

$3x^2-15x-2x+10=0$

$3x(x-5)-2(x-5)=0$

$(x-5)(3x-2)=0$

$x=5$ , $x=\frac{2}{3}$ ×

$[x=5]$

Case-2:

$x^2-4x+2<0$

$(x-(2+\sqrt{2}))(x-(2-\sqrt{2}))<0$

$x \in (2-\sqrt{2}, 2+\sqrt{2})$

$3(-x^2+4x-2)-5x+4=0$

$-3x^2+12x-6-5x+4=0$

$-3x^2+7x-2=0$

$3x^2-7x+2=0$

$3x^2-6x-x+2=0$

$3x(x-2)-1(x-2)=0$

$(3x-1)(x-2)=0$

$[x=2]$ , $x=\frac{1}{3}$ ×

∪

$[x=2,5]$ Ans.

**Kriti Mathur Raj.**

KTK| $3|x^2-4x+2| = 5x-4$

Case ① $x^2-4x+2 \geq 0$ ↷ Case ②: $x^2-4x+2 < 0$ ↷

$(x-(2+\sqrt{2}))(x-(2-\sqrt{2})) \geq 0$

$x \in (-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty)$

$x \in (2-\sqrt{2}, 2+\sqrt{2})$

$3x^2-12x+6-5x+4=0$

$3x^2-17x+10=0$

$x = \frac{17 \pm 13}{6}$

$x = 5, \cancel{\frac{2}{3}}$ ✓

$-3x^2+12x-6-5x+4=0$

$3x^2-7x+2=0$

$x = \frac{7 \pm 5}{6}$ $(x-6)(x-1)=0$

$x=6, x=1.$

$x = 2, \cancel{\frac{1}{3}}$

∪

Ans = 2,5

sakshisahu

QUESTION

(KTK 2)

The number of real roots of the equation $x|x| - 5|x + 2| + 6 = 0$, is :

(A) 4

(B) 3

(C) 5

(D) 6

Ans. B

KTK-2.) The number of real roots of the equation

$$x|\overset{T_1}{x}| - 5|\overset{T_2}{x+2}| + 6 = 0$$

**Kriti Mathur Raj.**

| | | | | | |
|---|---|---|---|---|---|
| $T_1$ | −ve | | −ve | | +ve |
| $T_2$ | −ve | −2 | +ve | 0 | +ve |

Case-1

If $x \geq 0$

$x(x) - 5(x+2) + 6 = 0$

$x^2 - 5x - 10 + 6 = 0$

$x^2 - 5x - 4 = 0$

$$x = \frac{5 \pm \sqrt{25 - 4(-4)}}{2}$$

$$x = \frac{5 \pm \sqrt{41}}{2}$$

$\left[x = \frac{5+\sqrt{41}}{2}\right]$, $\frac{5-\sqrt{41}}{2}$ ×

Case-2

If $-2 < x < 0$

$x(-x) - 5(x+2) + 6 = 0$

$-x^2 - 5x - 10 + 6 = 0$

$-x^2 - 5x - 4 = 0$

$x^2 + 5x + 4 = 0$

$x^2 + 4x + x + 4 = 0$

$x(x+4) + 1(x+4) = 0$

$(x+1)(x+4) = 0$

$[x = -1]$, $x = -4$ ×

Case-3

If $x \leq -2$

$x(-x) - 5(-x-2) + 6 =$

$-x^2 + 5x + 16 = 0$

$x^2 - 5x - 16 = 0$

$$x = \frac{5 \pm \sqrt{25 - 4(-16)}}{2}$$

$$x = \frac{5 \pm \sqrt{89}}{2}$$

$\left[x = \frac{5-\sqrt{89}}{2},\right]$ $\frac{5+\sqrt{}}{2}$

No. of real roots = $\underline{\underline{3}}$

KTK 02 The number of Real roots of the equation $x|x| - 5|x+2| + 6 = 0$, is:

let assume to be $\underset{t_1}{x|x|} - 5\underset{t_2}{|x+2|} + 6 = 0$

|  |  |  |  |  |  |
|---|---|---|---|---|---|
| $t_1$ | -ve |  | ~~+ve~~ -ve |  | +ve |
| $t_2$ | -ve | -2 | +ve | 0 | +ve |

Case ①  $x \geq 0$

$x^2 - 5x - 10 + 6 = 0$

$\Rightarrow x^2 - 5x - 4 = 0$

$\Rightarrow x = \frac{5 \pm \sqrt{25+16}}{2}$

$b^2 - 4ac$
$= 25 - 4 \cdot 1 (4)$

$\Rightarrow x = 5 \pm \frac{\sqrt{41}}{2} \Rightarrow x = \left(\frac{5+\sqrt{41}}{2}\right), \left(5 \frac{-\sqrt{41}}{2}\right)$

$\cap \Rightarrow x = \frac{5+\sqrt{41}}{2}$

Case (II)

$-2 < x < 0$

$-x^2 - 5x - 10 + 6 = 0$

$\Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow x = \frac{-5 \pm \sqrt{25-16}}{2} \Rightarrow x = \frac{-5+3}{2}, \frac{-5-3}{2}$

$x = -1, -4$

C $x = -1$

Case (III)

$x \leqslant -2$

$-x^2 + 5x + 10 + 6 = 0$

$\Rightarrow x^2 - 5x - 16 = 0$

$\Rightarrow x = \frac{5 \pm \sqrt{89}}{2}$

$x = \frac{5+\sqrt{89}}{2}, \frac{5-\sqrt{89}}{2}$

C $x = \frac{5-\sqrt{89}}{2}$

∴ No. of Real solutions ~~are~~ is ③

**QUESTION**

**KTK 03**

Solve for x :

$$\log 4+\left(1+\frac{1}{2x}\right)\log 3=\log\left(\sqrt[x]{3}+27\right)$$

Ans. $x \in \phi$

3. $\log 4 + \left(1+\frac{1}{2x}\right)\log 3 = \log\left(\sqrt[x]{3}+27\right)$

Sol$^n$ – $\log 4 + \log 3 + \log 3^{1/2x} = \log\left(\sqrt[x]{3}+27\right)$

$$\cancel{\log}\left(12\cdot 3^{1/2x}\right) = \cancel{\log}\left(\sqrt[x]{3}+27\right)$$

$$12t = t^2+27$$

$$t^2-12t+27=0$$

$$(t-3)(t-9)=0$$

$3^{1/2x} = 3$

$\frac{1}{2x} = 1$

$2x = 1$

$x = \frac{1}{2}$

$x \geq 2,\ x \in N$

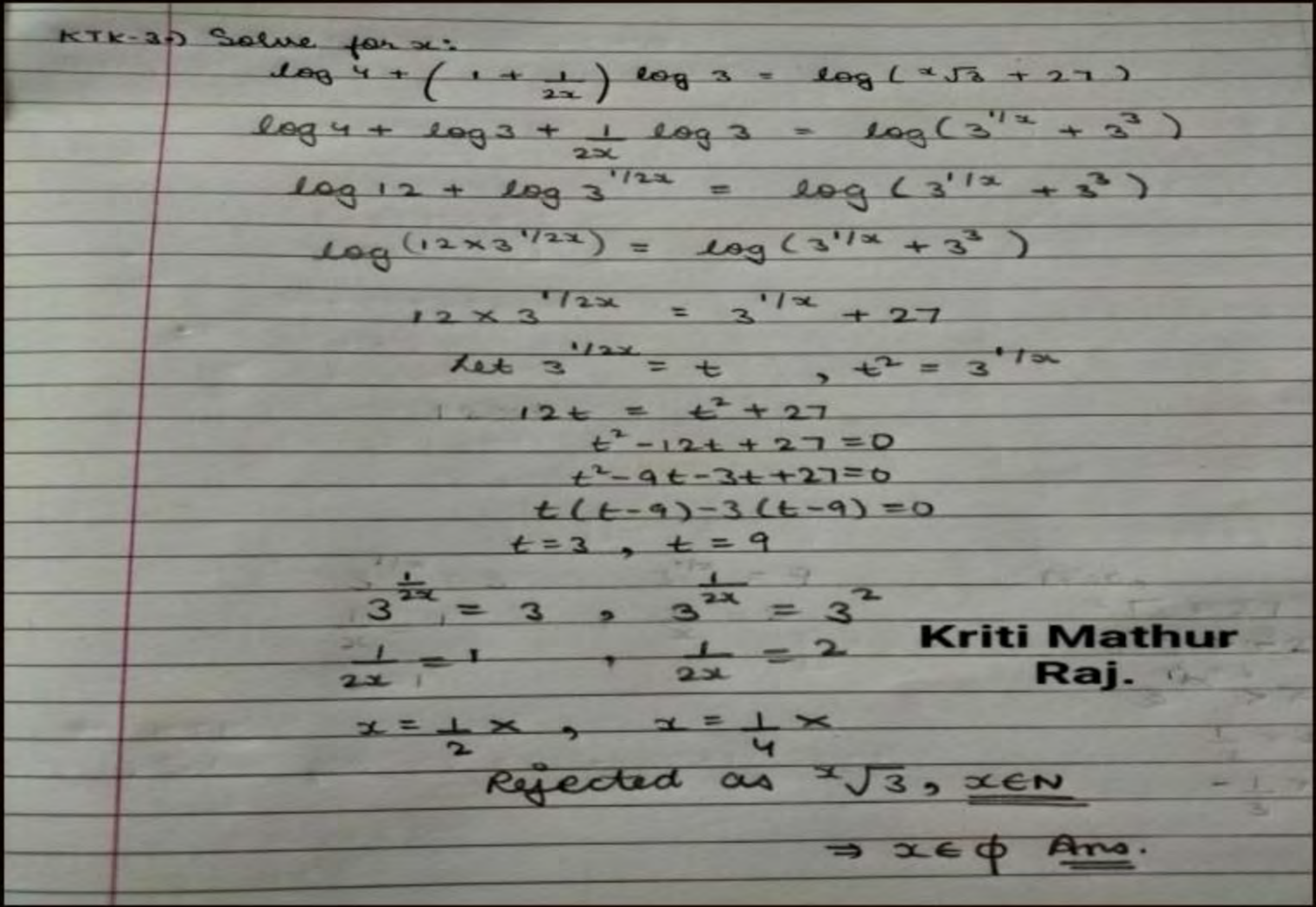
so,

$$\boxed{x \in \phi}$$

$3^{1/2x} = 3^2$

$\frac{1}{2x} = 2$

$x = \frac{1}{4}$



KTK-3) Solve for $x$:

$$\log 4 + \left(1 + \frac{1}{2x}\right)\log 3 = \log\left(\sqrt[x]{3} + 27\right)$$

$$\log 4 + \log 3 + \frac{1}{2x}\log 3 = \log\left(3^{1/x} + 3^3\right)$$

$$\log 12 + \log 3^{1/2x} = \log\left(3^{1/x} + 3^3\right)$$

$$\log\left(12\times 3^{1/2x}\right) = \log\left(3^{1/x} + 3^3\right)$$

$$12\times 3^{1/2x} = 3^{1/x} + 27$$

Let $3^{1/2x} = t$ , $t^2 = 3^{1/x}$

$$12t = t^2 + 27$$

$$t^2 - 12t + 27 = 0$$

$$t^2 - 9t - 3t + 27 = 0$$

$$t(t-9) - 3(t-9) = 0$$

$$t = 3 , \quad t = 9$$

$$3^{\frac{1}{2x}} = 3 , \quad 3^{\frac{1}{2x}} = 3^2$$

$$\frac{1}{2x} = 1 , \quad \frac{1}{2x} = 2$$

$$x = \frac{1}{2} \times , \quad x = \frac{1}{4} \times$$

Rejected as $\sqrt[x]{3}$, $\underline{\underline{x \in N}}$

$\Rightarrow x \in \phi$ Ans.

**Kriti Mathur**
**Raj.**

QUESTION

KTK 04

If $(x_1, y_1)$ and $(x_2, y_2)$ are the solution of the system of equation.

$\log_{225}(x) + \log_{64}(y) = 4$

$\log_x(225) - \log_y(64) = 1,$

then show that the value of $\log_{30}(x_1 y_1 x_2 y_2) = 12.$

QUESTION

KTK 05

The sum of the roots of the equation $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is :

(A) $\log_2 14$

(B) $\log_2 11$

(C) $\log_2 12$

(D) $\log_2 13$

Ans. B

**QUESTION**

**KTK 06**

Evaluate $\dfrac{\left((64)^{\frac{1}{\log_5 8}}+2^{\frac{2}{\log_{\sqrt{5}} 2}}\right)\left((\sqrt{11})^{\frac{2}{\log_{25} 11}}-(64)^{\log_8 \sqrt{5}}\right)}{300}$

Ans. 2

6. $$\frac{\left((64)^{\frac{1}{\log_5 8}} + 2^{\frac{2}{\log_{\sqrt{5}} 2}}\right)\left((\sqrt{11})^{\frac{2}{\log_{25} 11}} - (64)^{\log_8 \sqrt{5}}\right)}{300}$$

Sol$^n$ - $$\frac{\left((8)^{2\log_8 5} + 2^{2\log_2 \sqrt{5}}\right)\left(11^{\frac{1}{2}\times 2\log_{11} 25} - 8^{2\log_8 \sqrt{5}}\right)}{300}$$

$$= \frac{\left(25 + \sqrt{5}^2\right)\left(25 - \overset{5}{\cancel{25}}\right)}{300} = \boxed{\cancel{2}}$$

$$= \frac{625 - 25}{300} = \frac{6\cancel{20}}{\cancel{30}\cancel{0}} = \cancel{\frac{31}{15}}$$

$$= \frac{600}{300} = \boxed{2}$$

KTK-6.) Evaluate $\dfrac{\left((64)^{\frac{1}{\log_5 8}} + 2^{\frac{2}{\log_{\sqrt5} 2}}\right)\left((\sqrt{11})^{\frac{2}{\log_{25} 11}} - (64)^{\log_8 \sqrt5}\right)}{300}$

$$\Rightarrow \frac{\left((64)^{\log_8 5} + 2^{2\log_2 \sqrt5}\right)\left((\sqrt{11})^{2\log_{11} 25} - (64)^{\log_8 \sqrt5}\right)}{300}$$

$$\Rightarrow \frac{\left(5^{\log_8 64} + \sqrt5^{2\log_2 2}\right)\left(25^{2\log_{11}(11)^{1/2}} - \sqrt5^{\log_8 64}\right)}{300}$$

$$\Rightarrow \frac{(5^2 + 5)(25 - 5)}{300} = \frac{(25+5)(25-5)}{300}$$

**Kriti Mathur**

$$= \frac{625 - 25}{300} = \frac{6\cancel{00}}{3\cancel{00}}$$

**Raj.**

$$= \underline{\underline{2}} \text{ Ans.}$$

**QUESTION**

**KTK 07**

Simplify : $5^{\log_{\frac{1}{5}}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}}\frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2}\frac{1}{10+2\sqrt{21}}$

Ans. 6

9. $5^{\log_{\frac{1}{5}}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}} \frac{4}{\sqrt{7}+\sqrt{3}} + \log_{\frac{1}{2}} \frac{1}{10+2\sqrt{21}}$

$= 2 + \log_2 \left(\frac{4}{\sqrt{7}+\sqrt{3}}\right)^2 + \log_2 10+2\sqrt{21}$

$= 2 + \log_2 \left(\frac{16}{10+2\sqrt{21}}\right) + \log_2 \left(10+2\sqrt{21}\right)$

$= 2 + \log_2 \left(\frac{16}{10+2\sqrt{21}} \times 10+2\sqrt{21}\right) = 2 + \log_2 16$

$\boxed{6}$ ✓

KTK-9.) Simplify : $5^{\log_{\frac{1}{5}}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}} \frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2} \frac{1}{10+2\sqrt{21}}$

$$= 5^{\log_{5^{-1}} 2^{-1}} + \log_{2^{1/2}} \frac{4}{\sqrt{7}+\sqrt{3}} + \log_{2^{-1}} \frac{1}{10+2\sqrt{21}}$$

$$= 5^{\log_5 2} + 2\log_2 \frac{4}{\sqrt{7}+\sqrt{3}} - \log_2 \frac{1}{10+2\sqrt{21}}$$

$$= 2 + \log_2\left(\frac{4}{\sqrt{7}+\sqrt{3}}\right)^2 - \log_2\left(\frac{1}{10+2\sqrt{21}}\right)$$

$$= 2 + \log_2\left(\frac{\left(\frac{4}{\sqrt{7}+\sqrt{3}}\right)^2}{\frac{1}{10+2\sqrt{21}}}\right)$$

**Kriti Mathur**
**Raj.**

$$= 2+\log_2\left(\frac{16}{(\sqrt{7}+\sqrt{3})^2} \times 10+2\sqrt{21}\right)$$

$$= 2+\log_2\left(\frac{16}{\cancel{7+3+2\sqrt{21}}} \times \cancel{10+2\sqrt{21}}\right)$$

$$= 2+\log_2 2^4 \Rightarrow 2+4 = \underline{\underline{6}}$$ Ans.

# Solution to Previous TAH

## QUESTION [JEE Mains 2024 (8 April)]

TAH 11

The number of distinct real roots of the equation $|x+1|\,|x+3| - 4|x+2| + 5 = 0$ is

Ans. 2

The number of distinct real roots of the equation |x + 1| |x + 3| − 4|x + 2| + 5 = 0 is

$$\Rightarrow \underset{t_1}{|x+1|}\,\underset{t_2}{|x+3|} - 4\underset{t_3}{|x+2|} + 5 = 0$$

Redo

**Case-1** $x \leq -3$

$$(x+1)(x+3) + 4(x+2) + 5 = 0$$

$$x^2 + 4x + 3 + 4x + 8 + 5 = 0$$

$$x^2 + 8x + 16 = 0$$

$$(x+4)^2 = 0$$

$$\boxed{x = -4}$$

**Case-2**

$$-3 < x \leq -2$$

$$-(x+1)(x+3) + 4(x+2) + 5 = 0$$

$$-\left[x^2 + 4x + 3\right] + 4x + 8 + 5 = 0$$

$$-x^2 - 4x - 3 + 4x + 13 = 0$$

$$0 = x^2 - 10$$

$x = +\sqrt{10}$ or $-\sqrt{10}$ ⇒ both rejected

Ans. 2

Made with Goodnotes

**Case-3** $-2 < x \le -1$

$$-(x+1)(x+3) - 4(x+2) + 5 = 0$$

$$-x^2 - 4x - 3 - 4x - 8 + 5 = 0$$

$$x^2 + 8x - 6 = 6$$

$$x = \frac{-8 \pm \sqrt{64 + (4)(1)(+6)}}{2}$$

$$= \frac{-8 \pm \sqrt{88}}{2}$$

$$= -4 \pm 2\sqrt{22}$$

↓

rejected

**Case-4** $x > -1$

$$(x+1)(x+3) - 4(x+2) + 5 = 0$$

$$x^2 + 4x + 3 - 4x - 8 + 5 = 0$$

$$x^2 = 0$$

$$\boxed{x = 0}$$

Union of all the cases

give $\boxed{x = 0 \;\&\; -4}$

②

Made with Goodnotes

QUESTION

The number of solutions of the equation
$\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0, x>0$, is :

Ans. 1

Q). The number of solutions of the equation

$$\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0,\ x>0\text{, is:}$$

$$\log_{(x+1)}[(2x+5)(x+1)]+\log_{(2x+5)}(x+1)^2 \quad -4=0$$

$$\log_{(x+1)}(2x+5)+\log_{(x+1)}(x+1)+2\log_{(2x+5)}(x+1)-4=0$$

$$\text{Let} = \log_{(x+1)}(2x+5)=t$$

TAH-01

$$t+1+\frac{2}{t}-4=0$$

$\boxed{\text{1 Solution}}$

$$t^2-3t+2=0$$
$$t^2-t-2t+2=0$$
$$t(t-1)-2(t-1)=0$$
$$(t-1)(t-2)=0$$

$t=1$ , $t=2$

$$\log_{(x+1)}(2x+5)=1$$
$$2x+5=x+1$$
$$x=-4$$

$\boxed{\text{Put and chek}}$ X

$$\log_{(x+1)}(2x+5)=2$$
$$\cancel{2x}+5=x^2+1+\cancel{2x}$$
$$0=x^2-4$$
$$0=x^2-(2)^2$$
$$(x+2)(x-2)=0$$
$$x=2,-2$$

$\boxed{\text{Put and chek}}$

-2 Not Possible

---

Date __/__/___

L-14

TAH-01

The number of solutions of the equation $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0, x>0,$

$$\log_{(x+1)}(2x^2+7x+5)+2\log_{(2x+5)}(x+1)-4=0$$
$$\log_{(x+1)}((x+1)(2x+5))+2\log_{(2x+5)}(x+1)-4=0$$
$$\log_{(x+1)}(x+1)+\log_{(x+1)}(2x+5)+2\log_{(2x+5)}(x+1)-4=0$$
$$1+\log_{(x+1)}(2x+5)+\frac{2}{\log_{(x+1)}(2x+5)}-4=0$$

$$\text{let } \log_{(x+1)}(2x+5)=t$$

$$t+\frac{2}{t}-3=0$$
$$t^2-3t+2=0$$
$$(t-1)(t-2)=0$$
$$t=1,2$$

$$\log_{(x+1)}(2x+5)=1,\ 2$$

① $2x+5=(x+1)^1$ or $2x+5=(x+1)^2$

$x=-4$ ↓ Not possible

$2x+5=x^2+2x+1$

$x^2=4$

$x=\pm 2$

$x=2$ ✓, $-2$ ↳ Not possib

Only $\underline{\underline{x=2}}$ Ans.

## Lecture - 14

TAH1⇒ The number of solutions of the equation
$\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0$ , $x>0$ is:

Ans. $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2=4$

$2x^2+7x+5$
$2x^2+2x+5x+5$
$2x(x+1)+5(x+1)$

$\log_{(x+1)}(2x^2+7x+5)+2\log_{(2x+5)}(x+1)=4$

$\log_{(x+1)}(2x+5)(x+1)+2\log_{(2x+5)}(x+1)=4$

$=\log_{(x+1)}(x+1)+\log_{(x+1)}(2x+5)+\dfrac{2}{\log_{(x+1)}(2x+5)}=4$

$\log_{(x+1)}(2x+5)+\dfrac{2}{\log_{(x+1)}(2x+5)}=3$

Let $\log_{(x+1)}(2x+5)=t$

**Kriti Mathur Raj.**

$t+\dfrac{2}{t}=3 \Rightarrow t^2+2=3t$
$\Rightarrow t^2-3t+2=0$
$\Rightarrow t^2-2t-t+2=0$
$\Rightarrow t(t-2)-1(t-2)=0$
$\Rightarrow (t-1)(t-2)=0$

$t=1$
$\log_{(x+1)}(2x+5)=1$
$2x+5=x+1$
$x+5-1=0$
$x=-4$ ×
as $x>0$

$t=2$
$\log_{(x+1)}(2x+5)=2$
$(2x+5)=(x+1)^2$
$x^2+1+\cancel{2x}-\cancel{2x}-5=0$
$x^2-4=0$
$(x+2)(x-2)=0$
$\boxed{x=2}$ , $x=-2$ ×

No. of solutions = 1

DATE
PAGE No.
Gyan

Tah1

$\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2-4=0$ $\quad x>0$

$\log_{(x+1)}(2x^2+5x+2x+5)+\log_{(2x+5)}(x+1)^2-4=0$

$\log_{(x+1)}(x+1)(2x+5)+\log_{(2x+5)}(x+1)^2-4=0$

$\log_{(x+1)}(x+1)+\log_{(x+1)}(2x+5)+2\log_{(2x+5)}(x+1)-4=0$

let $\log_{(x+1)}(2x+5)=t$

sakshisahu

$$1+t+\frac{2}{t}-4=0$$

$$t^2+2-3t=0$$

$$t^2-3t+2=0$$

$$(t-2)(t-1)=0$$

$$t=1,2$$

only one Sol^n $x=\underline{2}$

$\log_{(x+1)}(2x+5)=1\,,\,2$

$\underline{(2x+5)=(x+1)}$

$x=-4$

✗ rejected

$\underline{2x+5=(x+1)^2}$

$\cancel{2x}+5=x^2+1+\cancel{2x}$

$4=x^2$

$x=\pm 2$

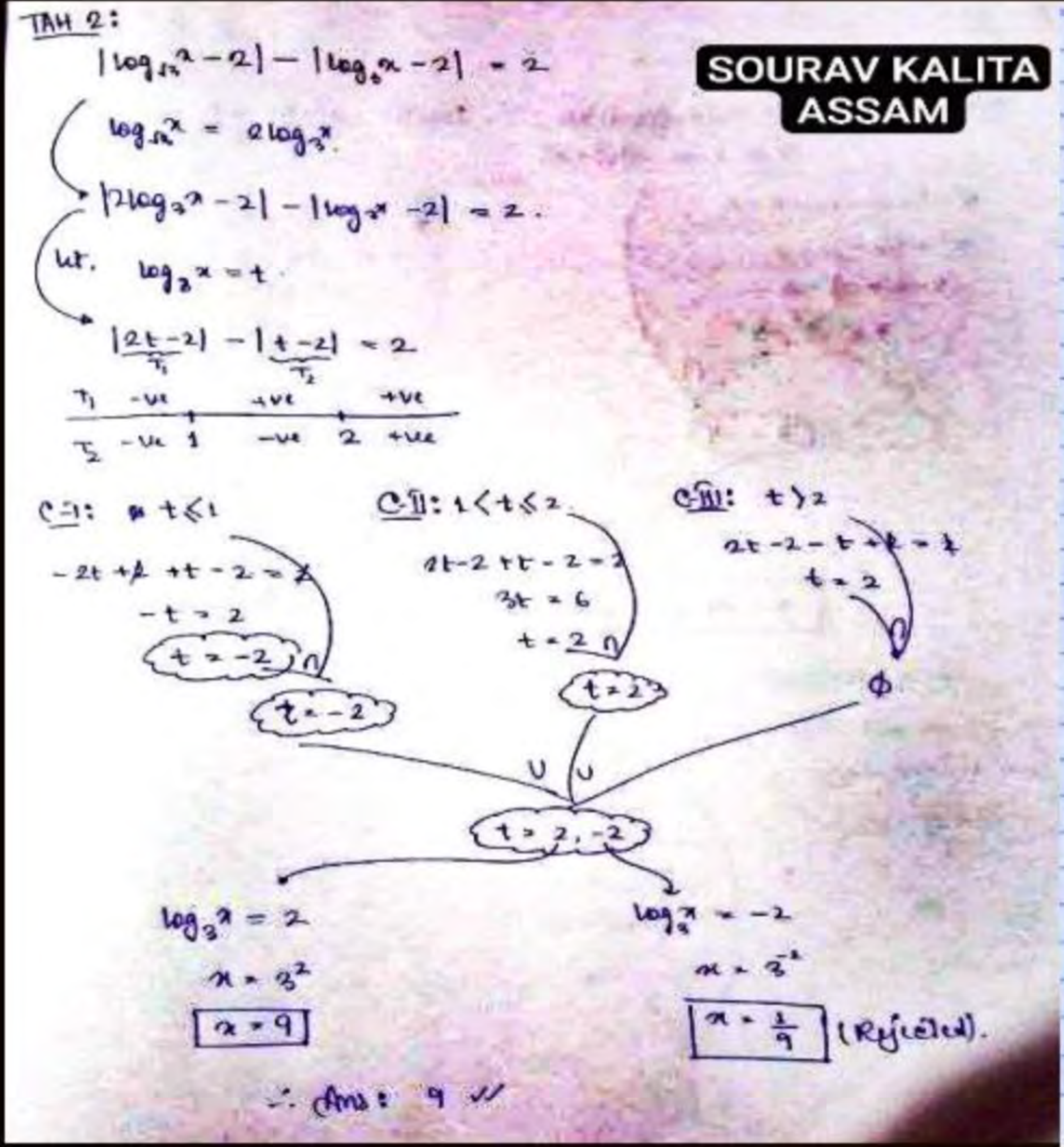
$x=2,-\cancel{2}$

reject

**Question**

Find the integral value of x satisfying the equation $\left|\log_{\sqrt{3}} x - 2\right| - |\log_3 x - 2| = 2$.

[Ans. 9]



SOURAV KALITA
ASSAM

TAH 2:

$|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$

$\log_{\sqrt{3}} x = 2\log_3 x$

$|2\log_3 x - 2| - |\log_3 x - 2| = 2.$

let. $\log_3 x = t$

$|\underbrace{2t-2}_{T_1}| - |\underbrace{t-2}_{T_2}| = 2$

| | | | | | |
|---|---|---|---|---|---|
| $T_1$ | -ve | | +ve | | +ve |
| $T_2$ | -ve | 1 | -ve | 2 | +ve |

C-I: $t \leq 1$

$-2t + \cancel{2} + t - 2 = \cancel{2}$

$-t = 2$

$t = -2$ $\cap$

$t = -2$

C-II: $1 < t \leq 2$

$2t - 2 + t - 2 = 2$

$3t = 6$

$t = 2$ $\cap$

$t = 2$

C-III: $t > 2$

$2t - 2 - t + \cancel{2} = \cancel{2}$

$t = 2$

$\cap$

$\phi$

$\cup$ $\cup$

$t = 2, -2$

$\log_3 x = 2$

$x = 3^2$

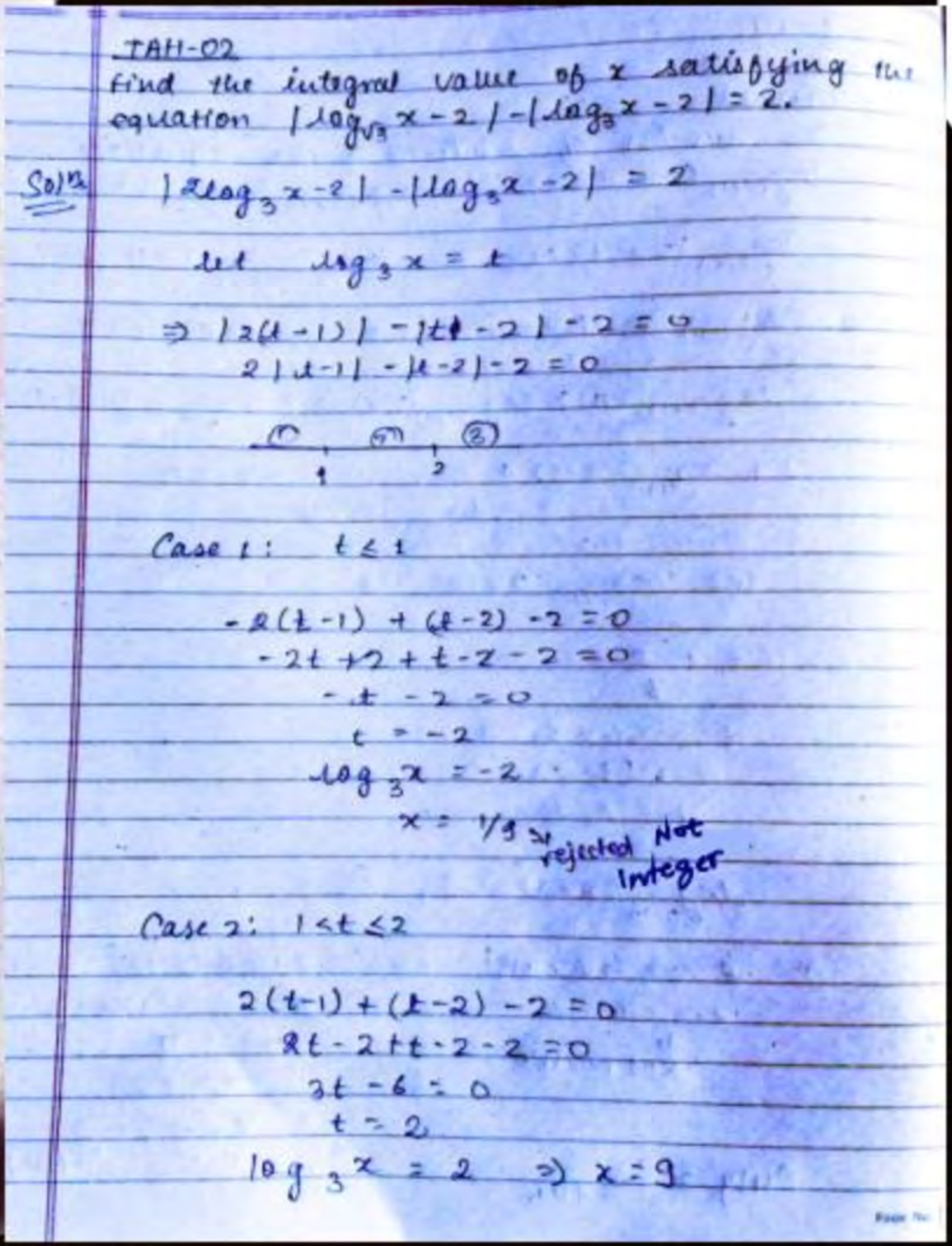
$\boxed{x = 9}$

$\log_3 x = -2$

$x = 3^{-2}$

$\boxed{x = \frac{1}{9}}$ (Rejected).

∴ Ans: 9 ✓



TAH-02

Find the integral value of $x$ satisfying the equation $|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2.$

Soln

$|2\log_3 x - 2| - |\log_3 x - 2| = 2$

let $\log_3 x = t$

$\Rightarrow |2(t-1)| - |t - 2| - 2 = 0$

$2|t-1| - |t-2| - 2 = 0$

① ② ③
1 2

Case 1: $t \leq 1$

$-2(t-1) + (t-2) - 2 = 0$

$-2t + 2 + t - 2 - 2 = 0$

$-t - 2 = 0$

$t = -2$

$\log_3 x = -2$

$x = 1/9$ ⇒ rejected Not Integer

Case 2: $1 < t \leq 2$

$2(t-1) + (t-2) - 2 = 0$

$2t - 2 + t - 2 - 2 = 0$

$3t - 6 = 0$

$t = 2$

$\log_3 x = 2 \Rightarrow x = 9$

TAH 2-) Find the integral value of x satisfying the equation

$$|\log_{\sqrt{3}}^{x} - 2| - |\log_{3}^{x} - 2| = 2$$

$$|\log_{3^{1/2}}^{x} - 2| - |\log_{3}^{x} - 2| = 2$$

$$\left|\frac{1}{\frac{1}{2}}\log_{3}^{x} - 2\right| - |\log_{3}^{x} - 2| = 2$$

$$|2\log_{3}^{x} - 2| - |\log_{3}^{x} - 2| = 2$$

**Kriti Mathur Raj.**

Let $\log_{3}^{x} = t \Rightarrow 2|\overset{T_1}{t-1}| - |\overset{T_2}{t-2}| = 2$

$$\begin{array}{c|c|c|c} T_1 & -ve & +ve & +ve \\ T_2 & -ve & -ve & +ve \\ & 1 & 2 & \end{array}$$

Case-1

If $x \geq 2$

$2(t-1) - (t-2) = 2$

$2t - 2 - t + \cancel{2} - \cancel{2} = 0$

$t - 2 = 0$

$[t = 2]$

Case-2

If $1 \leq x < 2$

$2(t-1) - (-t+2) = 2$

$2t - 2 + t - 2 - 2 = 0$

$3t - 8 = 0$

$t = \frac{8}{3}$ ×

Case-3

If $x < 1$

$2(-t+1) - (-t+2) - 2 = 0$

$-2t + \cancel{2} + t - \cancel{2} - 2 = 0$

$-t - 2 = 0$

$[t = -2]$

$t = 2, -2$

at $t = 2$

$\log_{3}^{x} = 2$

$[x = 9]$

at $t = -2$

$\log_{3}^{x} = -2$

$\left[x = \frac{1}{9}\right]$

Integral value of $x = \underline{\underline{9}}$

**QUESTION**

$$|x^2 + 4x + 3| + 2x + 5 = 0$$

SOURAV KALITA
ASSAM

## TAH 03:

$$|x^2+4x+3|+2x+5=0$$

**C-I:** $x^2+4x+3 \geq 0 \rightarrow (x+3)(x+1) \geq 0$

$x \in (-\infty,-3] \cup [-1,\infty)$.

$x^2+4x+3+2x+5=0$

$x^2+6x+8=0$

$(x+4)(x+2)=0$

$x=\cancel{-2},-4$

$\cap$

$x=-4$

**Case II:** $x^2+4x+3<0$

$\Rightarrow (x+3)(x+1)<0$

$x \in (-3,-1)$.

$-x^2-4x-3+2x+5=0$

$-x^2-2x+2=0$

$x^2+2x-2=0$

$D=4-4(2)$

$=12>0$

$$x=\frac{-2\pm\sqrt{12}}{2}$$

$x=-1\pm\sqrt{3}$

$\cap$

$x=-1-\sqrt{3}$

$\cup$

$\boxed{x=-4 \text{ or } -1-\sqrt{3}}$ ✓

TAH 3:) $|x^2+4x+3|+2x+5=0$

**Kriti Mathur**

**Raj.**

Case 1:

If $x^2+4x+3\ge 0$

$x^2+3x+x+3\ge 0$

$x(x+3)+1(x+3)\ge 0$

$(x+1)(x+3)\ge 0$

$x\in(-\infty,-3]\cup[-1,\infty)$

$x^2+4x+3+2x+5=0$

$x^2+6x+8=0$

$x^2+4x+2x+8=0$

$x(x+4)+2(x+4)=0$

$(x+2)(x+4)=0$

$x=-2,-4$ $\cap$

$[x=-4]$

Case-2:

If $x^2+4x+3<0$

$(x+1)(x+3)<0$

$x\in(-1,-3)$

$-x^2-4x-3+2x+5=0$

$-x^2-2x+2=0$

$x^2+2x-2=0$

$D=\sqrt{4-4(-2)}$

$D=\sqrt{12}=2\sqrt{3}$

$x=\frac{-2\pm 2\sqrt{3}}{2}$

$\cap$ $x=-1+\sqrt{3},-1-\sqrt{3}$

$[x=-1-\sqrt{3}]$

$\cup$

$[x=-4,-1-\sqrt{3}]$ Ans.

# Solution to Previous BPPs

## Lo Karo Duvaadaar Practice!!

1. $\log_5(x^2 - 3x + 3) > 0$

2. $\log_7[\log_5(x^2 - 7x + 15)] > 0$

3. $\log_{\left(\frac{1}{2}\right)}[\log_5(x^2 - 7x + 17)] > 0$

4. $\log_{\left(\frac{1}{2}\right)}(\log_5(\log_2(x^2 - 6x + 40))) > 0$

5. $\log_3[\log_5 \log_2(x^2 - 9x + 50)] > 0$

6. $\log_6\left(\frac{x-2}{6-x}\right) > 0$

7. $\log_{0.5}(x^2 - 5x + 6) > -1$

8. $\log_8(x^2 - 4x + 3) < 1$

9. $\log_{\left(\frac{1}{4}\right)}\left(\frac{35-x^2}{x}\right) \geq -\frac{1}{2}$

# Answers

1. $(-\infty, 1) \cup (2, \infty)$
2. $(-\infty, 2) \cup (5, \infty)$
3. $(3, 4)$
4. $(2, 4)$
5. $(-\infty, 3) \cup (6, \infty)$
6. $(4, 6)$
7. $(1, 4)$
8. $(-1, 5)$
9. $(-1, 0) \cup (5, \infty)$

1) $x^2 - 3x + 3 > 1$

$x^2 - 2x - x + 2 > 0$

$(x-2)(x-1) > 0$

$$\boxed{x \in (-\infty, 1) \cup (2, \infty)}$$

2.) $\log_5 (x^2 - 7x + 15) > 1 \quad \cap \quad x^2 - 7x + 15 > 0$

$x^2 - 7x + 15 > 5$

$x^2 - 2x - 5x + 10 > 0$

$(x-2)(x-5) > 0$

$$\boxed{x \in (-\infty, +2) \cup (5, \infty)}$$

$D = 49 - 60$

$\downarrow$

$D < 0$

$x^2 - 7x + 15 > 0 \;\; \forall \, x \in R$

4.) $\log_{1/2}\left(\log_5\left(\log_2\left(x^2-6x+40\right)\right)\right) > 0$

$\log_5(\log_2(x^2-6x+40)) < 1$

$\log_2(x^2-6x+40) < 5$

$x^2-6x+40 < 32$

$x^2-6x+8 < 0$

$x^2-4x-2x+8 < 0$

$(x-4)(x-2) < 0$

$x \in (2, 4) \rightarrow i$

$\log_5(\log_2(x^2-6x+40)) > 0$

$\log_2(x^2-6x+40) > 1$

$x^2-6x+40 > 2$

$x^2-6x+38 > 0$

$\downarrow$

$D<0, \; a>0$

$\downarrow$

$x^2-6x+38 > 0 \;\; \forall x \in R \rightarrow ii$

$\log_2 (x^2-6x+40) > 0$

$x^2-6x+40 > 1$

$x^2-6x+39 > 0$

$\downarrow$

$D<0, \ a>0$

$x^2-6x+39 > 0 \ \forall x \in R$

$x \in R \longrightarrow \text{iii}$

$x^2-6x+40 > 0$

$\downarrow$

$D<0, \ a>0$

$x^2-6x+40 > 0 \ \forall x \in R \rightarrow \text{iv}$

$\text{i} \cap \text{ii} \cap \text{iii} \cap \text{iv}$

$$\boxed{x \in (2,4)}$$

6.) $\log_6\left(\frac{x-2}{6-x}\right) > 0$ & $\frac{x-2}{6-x} > 0$

↳ No Need

$$\frac{x-2}{6-x} > 1$$

$$\frac{x-2}{6-x} - 1 > 0$$

$$\frac{x-2-(6-x)}{6-x} > 0$$

$$\frac{x-2-6+x}{6-x} > 0$$

$$\frac{2x-8}{x-6} < 0$$

$$\frac{x-4}{x-6} < 0$$

$$\boxed{x \in (4,6)}$$

# bpp - 6

$$\log_6\left(\frac{x-2}{6-x}\right) > 0$$

Solution:-

$$\frac{x-2}{6-x} > 1 \quad , \quad \frac{x-2}{6-x} > 0$$

$\downarrow$

No Need

$$\frac{x-2}{6-x} - 1 > 0$$

$$\frac{x-2-6+x}{6-x} > 0$$

$$\frac{2x-8}{6-x} > 0$$

$$\frac{2(x-4)}{6-x} > 0$$

$$\frac{x-4}{x-6} < 0$$

$$x \in (4,6)$$

8.) $\log_8 (x^2-4x+3) < 1$ & $x^2-4x+3>0$

$$x^2-4x+3 < 8$$

$$x^2-4x-5<0$$

$$(x-5)(x+1)<0$$

$$x \in (-1,5)$$

$$x^2-3x-x+3>0$$

$$(x-3)(x-1)>0$$

$$x \in (-\infty,1) \cup (3,\infty)$$

$\cap$

$$\boxed{x \in (-1,1) \cup (3,5)}$$

# THANK YOU